Application of the implicit MacCormack method to the computation of supersonic turbulent jets, using an algebraic stress model.

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Over the last years the significant progress has been achieved in the mathematical modelling of supersonic jets. The problem is solved on the basis of the complete Navier-Stokes equations [I,2],as well as with the usage of the simplified ("parabolized") Navier-Stokes equations[3,4,5].

For the jets calculation on the basis of the parabolized Navier-Stokes equations,as a rule,explicit numerical methods are used: in Refs. 3,4 -MacCormack method [6], in Ref.5 - specially developed explicit scheme. The major fault of the schemes is the limitation on the axial step size, that may lead to a significant loss of computer time.

At the beginning of the 80th MacCormack[7] proposed effective implicit method of solving the complete unsteady Navier-Stokes equations. The main merits of this method are: unconditional stability, absence of necessity in solving the equations with block tridiagonal matrices.

The present work presents the modification of the implicit MacCormack method, applicable for the solving parabolized Navier-Stokes equations in the process of the supersonic jet calculation.

The particular attention is given to the problem of turbulent mixing modelling.

I. The turbulent gas mixture flow equations are obtained by means of formal averaging of the unsteady Navier-Stokes equations. In this work two methods of the division of variables into mean and fluctuating components are used [8]:

I) $p=\bar{p}+p'$, where the line means an ensemble averaging;

2) $T=\tilde{T}+T''$, where $\tilde{T}=\overline{\rho T}/\bar{\rho}$ - mass averaged quantity.

The equations for a high Reynolds number axisymmetric jet, when the chemistry is frozen, are given below

$$\frac{\partial}{\partial x}(\bar{\rho}\tilde{u})+\frac{\partial}{\partial y}(\bar{\rho}\tilde{v})+\frac{\bar{\rho}\tilde{v}}{y}=0, \tag{1}$$

$$\bar{\rho}\tilde{u}\frac{\partial\tilde{u}}{\partial x}+\bar{\rho}\tilde{v}\frac{\partial\tilde{u}}{\partial y}=-\frac{\partial\bar{P}}{\partial x}-\frac{\partial}{\partial x}(\bar{\rho}\widetilde{u''^2})-\frac{\partial}{\partial y}(\bar{\rho}\widetilde{u''v''})-\frac{\bar{\rho}\widetilde{u''v''}}{y}, \tag{2}$$

$$\bar{\rho}\tilde{u}\frac{\partial\tilde{v}}{\partial x}+\bar{\rho}\tilde{v}\frac{\partial\tilde{v}}{\partial y}=-\frac{\partial\bar{P}}{\partial y}-\frac{\partial}{\partial x}(\bar{\rho}\widetilde{u''v''})-\frac{\partial}{\partial y}(\bar{\rho}\widetilde{v''^2})-\frac{\bar{\rho}(\widetilde{v''^2}-\widetilde{w''^2})}{y}, \tag{3}$$

$$\bar{\rho}\tilde{u}\frac{\partial\tilde{h}_0}{\partial x}+\bar{\rho}\tilde{v}\frac{\partial\tilde{h}_0}{\partial y}=-\frac{\partial}{\partial x}(\bar{\rho}\widetilde{u''h''_0})-\frac{\partial}{\partial y}(\bar{\rho}\widetilde{v''h''_0})-\frac{\bar{\rho}\widetilde{v''h''_0}}{y}, \tag{4}$$

$$\bar{\rho}\tilde{u}\frac{\partial\tilde{C}}{\partial x}+\bar{\rho}\tilde{v}\frac{\partial\tilde{C}}{\partial y}=-\frac{\partial}{\partial x}(\bar{\rho}\widetilde{u''C''})-\frac{\partial}{\partial y}(\bar{\rho}\widetilde{v''C''})-\frac{\bar{\rho}\widetilde{v''C''}}{y}, \tag{5}$$

$$\tilde{h}_0=\int_0^T C_p\,dT+0{,}5(\tilde{u}^2+\tilde{v}^2)+K, \tag{6}$$

$$\bar{p} = \bar{\rho} R \tilde{T} / M_{\Sigma}, \tag{7}$$

where x, y - orthogonal coordinate system (x - coordinate measured along the jet axis), $\tilde{u}, \tilde{v}$ - axial and radial velocity components, $\bar{\rho}$ - density, $\bar{p}$ - pressure, h_0 - specific total enthalpy, c - inert component concentration, R - universal gas constant, M_{Σ} - molecular weight of gas mixture, C_p - specific heat.

For the system closure it is necessary to apply any turbulence model. Many of the models used now include K and ε transport equations (K - turbulent kinetic energy, ε - rate of dissipation)

$$\bar{\rho}\tilde{u}\frac{\partial K}{\partial x} + \bar{\rho}\tilde{v}\frac{\partial K}{\partial y} = -\frac{\partial}{\partial x}\left(\bar{\rho}\widetilde{u''K}\right) - \frac{\partial}{\partial y}\left(\bar{\rho}\widetilde{v''K}\right) - \frac{\overline{\rho v'' K}}{y} + \bar{\rho}\mathcal{P} - \bar{\rho}\varepsilon + \Pi_K + æ, \tag{8}$$

$$\bar{\rho}\tilde{u}\frac{\partial \varepsilon}{\partial x} + \bar{\rho}\tilde{v}\frac{\partial \varepsilon}{\partial y} = -\frac{\partial}{\partial x}\left(\bar{\rho}\widetilde{u''\varepsilon}\right) - \frac{\partial}{\partial y}\left(\bar{\rho}\widetilde{v''\varepsilon}\right) - \frac{\overline{\rho \widetilde{v''\varepsilon}}}{y} + \frac{\varepsilon}{K}\left(C_{\varepsilon 1}\bar{\rho}\mathcal{P} - C_{\varepsilon 2}\bar{\rho}\varepsilon + \Pi_K + æ\right), \tag{9}$$

where $\mathcal{P} = -\left[\widetilde{u''^2}\frac{\partial \tilde{u}}{\partial x} + \widetilde{u''v''}\left(\frac{\partial \tilde{u}}{\partial y} + \frac{\partial \tilde{v}}{\partial x}\right) + \widetilde{v''^2}\frac{\partial \tilde{v}}{\partial y} + \frac{\widetilde{w''^2}\tilde{v}}{y}\right]$, $\Pi_K = \frac{\overline{\rho' u''}}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x} + \frac{\overline{\rho' v''}}{\bar{\rho}}\frac{\partial \bar{p}}{\partial y}$, $æ = \overline{p'\frac{\partial v_i''}{\partial x_i}}$,
$C_{\varepsilon 1}, C_{\varepsilon 2}$ - numerical coefficients, will be chosen below.

2. For the determination of turbulent correlations appearing in the set of equations the algebraic stress model is used. The main priciples of the model construction are stated in [8].

In this case the following assumptions are used.

I) In the input turbulent stress transport equations the terms including pressure gradient are omitted and it is supposed that a dissipation and production are in equilibrium.

2) Consider θ - the flow angle on inner mixing layer boundary and S, n - orthogonal "mixing-layer-oriented" coordinate system (θ - is the angle between S and X axes).

For the viscous terms it is assumed that

$$\partial/\partial S \ll \partial/\partial n, \quad V_s \gg V_n \tag{I0}$$

Such approach is more correct than widely used approximation

$$\partial/\partial x \ll \partial/\partial y, \quad u \gg v \tag{II}$$

3) The received turbulence model, when using it for incompressible subsonic flows, should coinside with the classical $K - \varepsilon$ model [9].

As a result we have got the following set of equations describing the turbulent axially symmetric gas jets

$$\frac{\partial}{\partial x}(\bar{\rho}\tilde{u}) + \frac{\partial}{\partial y}(\bar{\rho}\tilde{v}) + \frac{\bar{\rho}\tilde{v}}{y} = 0, \tag{I2}$$

$$\bar{\rho}\tilde{u}\frac{\partial \tilde{u}}{\partial x} + \bar{\rho}\tilde{v}\frac{\partial \tilde{u}}{\partial y} = -\frac{\partial}{\partial x}\left(\bar{p} + \bar{\rho}\widetilde{V_n''^2}\right) + \frac{1}{y\cos^2\theta}\frac{\partial}{\partial y}\left(y\mu_T\frac{\partial \tilde{u}}{\partial y}\right), \tag{I3}$$

$$\bar{\rho}\tilde{u}\frac{\partial \tilde{v}}{\partial x} + \bar{\rho}\tilde{v}\frac{\partial \tilde{v}}{\partial y} = -\frac{\partial}{\partial y}\left(\bar{p} + \bar{\rho}\widetilde{V_n''^2}\right) + \frac{1}{\cos^2\theta}\frac{\partial}{\partial y}\left(\mu_T\frac{\partial \tilde{v}}{\partial y}\right), \tag{I4}$$

$$\bar{\rho}\tilde{u}\frac{\partial \tilde{h}_0}{\partial x} + \bar{\rho}\tilde{v}\frac{\partial \tilde{h}_0}{\partial y} = \frac{1}{y\cos^2\theta}\frac{\partial}{\partial y}\left[y\mu_T\left(\frac{1}{Pr_T}\frac{\partial \tilde{h}}{\partial y} + \frac{\partial K}{\partial y} + \frac{\tilde{u}}{\cos^2\theta}\frac{\partial \tilde{u}}{\partial y}\right)\right], \tag{I5}$$

$$\bar{\rho}\tilde{u}\frac{\partial \tilde{c}}{\partial x} + \bar{\rho}\tilde{v}\frac{\partial \tilde{c}}{\partial y} = \frac{1}{y\cos^2\theta}\frac{\partial}{\partial y}\left(y\frac{\mu_T}{Pr_T}\frac{\partial \tilde{c}}{\partial y}\right), \tag{I6}$$

$$\bar{\rho}\tilde{u}\frac{\partial K}{\partial x} + \bar{\rho}\tilde{v}\frac{\partial K}{\partial y} = \frac{1}{y\cos^2\theta}\frac{\partial}{\partial y}\left(y\mu_T\frac{\partial K}{\partial y}\right) + \bar{\rho}\mathcal{P} - \bar{\rho}\varepsilon + \Pi_K + æ, \tag{I7}$$

$$\bar{\rho}\tilde{u}\frac{\partial\varepsilon}{\partial x}+\bar{\rho}\tilde{v}\frac{\partial\varepsilon}{\partial y}=\frac{1}{y\cos^2\theta}\frac{\partial}{\partial y}\left(y\frac{\mu_T}{\sigma_\varepsilon}\frac{\partial\varepsilon}{\partial y}\right)+\frac{\varepsilon}{K}\left(C_{\varepsilon 1}\bar{\rho}\mathcal{P}-C_{\varepsilon 2}\bar{\rho}\varepsilon+\pi_K+\text{æ}\right), \tag{18}$$

$$\tilde{h}=\int_0^{\tilde{T}}C_p\,dT,\quad \tilde{h}_0=\tilde{h}+0,5(\tilde{u}^2+\tilde{v}^2)+K,\quad \bar{p}=\bar{\rho}R\tilde{T}/M_\Sigma,$$

$$\mu_T=\frac{C_\mu}{0,52}\cdot\frac{\widetilde{V_n''^2}}{K}\cdot\bar{\rho}\,\frac{K^{2}}{\varepsilon},\quad \widetilde{V_n''}=\frac{2}{3}K\left[1-0,222\left(1-\frac{\text{æ}}{\bar{\rho}\varepsilon}\right)\right],$$

$$\bar{\rho}\mathcal{P}=\frac{\mu_T}{\cos^4\theta}\left(\frac{\partial\tilde{u}}{\partial y}\right)^2$$

The numerical coefficients will be as follows

$$Pr_T=0,7,\quad C_{\varepsilon 1}=1.43,\quad C_{\varepsilon 2}=\begin{cases}1.92, & \text{when } \theta\neq 0,\\ 1.92-0,0667f, & \text{when } \theta=0,\end{cases}\quad C_\mu=\begin{cases}0,09, & \text{when } \theta\neq 0,\\ 0,09-0.04f, & \theta=0,\end{cases}$$

$$\sigma_\varepsilon=1.3,$$

where $f=\left|\frac{y_G}{2\Delta u}\left(\frac{d\tilde{u}_0}{dx}-\left|\frac{d\tilde{u}_0}{dx}\right|\right)\right|^{0,2}$

$\tilde{u}_0$ - value of centerline velocity, Δu - a velocity difference within the mixing layer, y_G -a mixing layer width (see[9])

Parameters enterring into the turbulence model—æ (a correlation of velocity divergence and pressure fluctuations) and π_K – appear only in case of high speed flows.

The parameter æ can be modelled as follows

$$\text{æ}=-\bar{\rho}\varepsilon\left(C_A M_T+\tilde{C}_A M_T^2\right), \tag{19}$$

where $M_T=\sqrt{K}/a$, a - sound speed, C_A, $\tilde{C}_A$ -numerical constants (C_A =0.3, $\tilde{C}_A$ =7.5)

The æ parameter presence reduces the turbulent mixing intensity and increases the turbulence anisotropy. These facts are corroborated by some experiments (see Refs.10,11)

The π_K parameter is by the expression of Ref. 12

$$\pi_K=C_\pi\frac{K}{a^2}\tilde{u}\frac{\partial\bar{p}}{\partial x}\frac{1}{\cos^2\theta}, \tag{20}$$

where $C_\pi=2.5$

3. The system of equations (12)-(18) in a conservative form has the following form

$$\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}+H=Y^{-1}\frac{\partial}{\partial y}\left(YL\frac{\partial\Psi}{\partial y}\right)+S, \tag{21}$$

where $F=\left(\rho u,\rho u^2+p+\rho\widetilde{V_n''^2},\rho uv,\rho uh_0,\rho uc,\rho u\varepsilon,\rho uK\right)^T,$

$G=\left(\rho uv,\rho uv,\rho v^2+p+\rho\widetilde{V_n''^2},\rho vh_0,\rho vc,\rho v\varepsilon,\rho vK\right)^T,$

$H=\frac{1}{y}\left(\rho v,\rho uv,\rho v^2,\rho vh_0,\rho vc,\rho v\varepsilon,\rho vK\right)^T,\quad \Psi=\left(\rho,u,v,h,c,\varepsilon,K\right)^T,$

$$Y=\begin{bmatrix}1&0&0&0&0&0&0\\0&\bar{y}&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&\bar{y}&0&0&0\\0&0&0&0&\bar{y}&0&0\\0&0&0&0&0&\bar{y}&0\\0&0&0&0&0&0&\bar{y}\end{bmatrix},\quad L=\begin{bmatrix}0&0&0&0&0&0&0\\0&\mu_T&0&0&0&0&0\\0&0&\mu_T&0&0&0&0\\0&\frac{u\mu_T}{\cos^2\theta}&0&\frac{\mu_T}{Pr_T}&0&0&\mu_T\\0&0&0&0&\mu/Pr_T&0&0\\0&0&0&0&0&\mu_T/\sigma_\varepsilon&0\\0&0&0&0&0&0&\mu_T\end{bmatrix}\cdot\frac{1}{\cos^2\theta},$$

$$S=\left[0,0,0,0,0,\frac{\varepsilon}{K}\left(C_{\varepsilon 1}\rho\mathcal{P}-C_{\varepsilon 2}\rho\varepsilon+\pi_K+\text{æ}\right),\rho\mathcal{P}+\pi_K+\text{æ}-\rho\varepsilon\right]^T,\quad \bar{x}=\frac{x}{R_a},\ \bar{y}=\frac{y}{R_a}$$

The averaging signs are omitted in Eq.2I where it is possible.

The system is solved in the mapped coordinate system utilizing the simple restangular transformation [3]:

$$\xi = \bar{x}, \quad \eta = \eta(\bar{x}, \bar{y}) \tag{22}$$

The transformed form of the Eq.2I is

$$\frac{\partial \bar{F}}{\partial \xi} + \frac{\partial \bar{G}}{\partial \eta} + \bar{H} = Y^{-1}\frac{\partial}{\partial \eta}\left(\bar{L}\frac{\partial \Psi}{\partial \eta}\right) + \bar{S}, \tag{23}$$

where $\bar{F} = F/b$, $\bar{G} = G + \bar{a}F/b$, $\bar{H} = HR_a/b$, $\bar{L} = YLb/R_a$, $\bar{S} = \frac{SR_a}{b}$, $\bar{a} = \frac{\partial \eta}{\partial \bar{x}}$, $b = \frac{\partial \eta}{\partial \bar{y}}$
R_a - jet exit radius.

The splitting method was applied for solving the system.

At every axial step ($\xi^n \le \xi \le \xi^{n+1}$) sequence of equations is solved

$$\frac{\partial \tilde{F}}{\partial \xi} + \frac{\partial \bar{G}}{\partial \eta} + \bar{H} = Y^{-1}\frac{\partial}{\partial \eta}\left(\bar{L}\frac{\partial \Psi}{\partial \eta}\right), \tag{24}$$

$$\frac{\partial \tilde{\tilde{F}}}{\partial \xi} = \bar{S}, \tag{25}$$

on the condition that $\tilde{F}(\xi^n, \eta) = \bar{F}(\xi^n, \eta)$,
$\tilde{\tilde{F}}(\xi^n, \eta) = \tilde{F}(\xi^{n+1}, \eta)$,
where it is supposed $\bar{F}(\xi^{n+1}, \eta) = \tilde{\tilde{F}}(\xi^{n+1}, \eta)$

3.I: For the solving of Eq.24 a special modification of the MacCormack implicit method is applied; it is described in the author's work [I3] and it has all the merits of the MacCormack original method.

Predictor:

$$\Delta\tilde{F}_K^n = \Delta\xi\left\{\frac{\bar{G}_K^n - \bar{G}_{K+1}^n}{\Delta\eta} - \bar{H}_K^n + \left(Y^{-1}\right)_K^n \frac{1}{2\cdot\Delta\eta^2}\left[\left(\bar{L}_K^n + \bar{L}_{K+1}^n\right)\left(\Psi_{K+1}^n - \Psi_K^n\right) - \left(\bar{L}_K^n + \bar{L}_{K-1}^n\right)\left(\Psi_K^n - \Psi_{K-1}^n\right)\right]\right\}, \tag{26}$$

$$\left(E + \frac{\Delta\xi}{\Delta\eta}|\Omega|_K^n + \Delta\xi|\Phi|_K^n\right)\delta\tilde{F}_K^{\overline{n+1}} = \Delta\tilde{F}_K^n + \frac{\Delta\xi}{\Delta\eta}|\Omega|_{K+1}^n\,\delta\tilde{F}_{K+1}^{\overline{n+1}}, \tag{27}$$

$$\tilde{F}_K^{\overline{n+1}} = \tilde{F}_K^n + \delta\tilde{F}_K^{\overline{n+1}}. \tag{28}$$

Corrector:

$$\Delta\tilde{F}_K^{\overline{n+1}} = \Delta\xi\left\{\frac{\bar{G}_{K-1}^{\overline{n+1}} - \bar{G}_K^{\overline{n+1}}}{\Delta\eta} - \bar{H}_K^{\overline{n+1}} + \left(Y^{-1}\right)_K^{n+1}\frac{1}{2\Delta\eta^2}\left[\left(\bar{L}_K^{\overline{n+1}} + \bar{L}_{K+1}^{\overline{n+1}}\right)\left(\Psi_{K+1}^{\overline{n+1}} - \Psi_K^{\overline{n+1}}\right) - \left(\bar{L}_K^{\overline{n+1}} + \bar{L}_{K-1}^{\overline{n+1}}\right)\left(\Psi_K^{\overline{n+1}} - \Psi_{K-1}^{\overline{n+1}}\right)\right]\right\}, \tag{29}$$

$$\left(E + \frac{\Delta\xi}{\Delta\eta}|\Omega|_K^{\overline{n+1}} + \Delta\xi|\Phi|_K^{\overline{n+1}}\right)\delta\tilde{F}_K^{n+1} = \Delta\tilde{F}_K^{\overline{n+1}} + \frac{\Delta\xi}{\Delta\eta}|\Omega|_{K-1}^{\overline{n+1}}\,\delta\tilde{F}_{K-1}^{n+1}, \tag{30}$$

$$\tilde{F}_K^{n+1} = \frac{1}{2}\left(\tilde{F}_K^n + \tilde{F}_K^{\overline{n+1}} + \delta\tilde{F}_K^{n+1}\right). \tag{3I}$$

Matrices $|\Omega|$ and $|\Phi|$ have positive eigenvalues and are related to the Jacobians $\Omega = \partial\bar{G}/\partial\bar{F}$, $\Phi = \partial\bar{H}/\partial\bar{F}$; E -identity matrix.

After each step the physical variables ρ, u, v, p, c, K, ε must be determinated from the conservation variables.

It should be mentioned that this determination and correspondingly the matrices $|\Omega|$ and $|\Phi|$ constructing depends upon the jet velocity at the examining point: supersonic or subsonic.

When calculating subsonic regions in order to permit spatial marching the pressure should be determined not according to the vector F components, but according to the imposed streamwise gradient $\frac{\partial p}{\partial \xi}$. The flow parameters u, v, c, h, K,

$\mathcal{E}$ are determined from the F components, ρ - according to the continuity equation (see Refs. 3,4).

Let us consider first of all the determination of $|\Omega|$ and $|\Phi|$ for supersonic regions.

The matrix $|\Omega|$ is diagonalized as $|\Omega| = \sigma^{-1} D \sigma$, (32)

where σ is the eigenvector matrix of Ω and σ^{-1} is the inverse of σ.

$$d_j = max\{0.5(|\omega_j| + \frac{2\ell}{\Delta\eta} - \frac{\Delta\eta}{\Delta\xi}), 0\},\ |\omega_j| = \{|b\lambda_j + \bar{a}| + b\lambda_G + |\bar{a}|\lambda_F\} N/N^*,\ \alpha = 0.5(u^2+v^2),$$

$$\lambda_{1,2,5,6,7} = v/u,\ \lambda_{3,4} = (uv \mp aw)/N,\ w = \sqrt{u^2+v^2-a^2},\ N = u^2 - a^2, N^* = N - \gamma \tilde{V}_k'',\ \lambda_F = \frac{\gamma \tilde{V}_k''}{N}$$

$$\lambda_G = 0.5\left[\left|\frac{\beta v K}{uN}\right| + \sqrt{\left(\frac{\beta v K}{uN}\right)^2 + 4\beta \tilde{V}_k''(K + \tilde{V}_k'')/(Nu^2)}\ \right],\ \beta = \gamma - 1,$$

γ - is the ratio of specific heats. The matrix σ^{-1} is defined as

$$\sigma^{-1} = \begin{pmatrix} -\beta & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & \beta u & (av+uw)/w & (uw-av)/w & 0 & 0 & 0 \\ 0 & \beta v & (vw-au)/w & (au+vw)/w & 0 & 0 & 0 \\ b_{41} & b_{42} & I_{s1} & I_{s1} & 0 & 0 & 0 \\ 0 & 0 & f & f & 1 & 0 & 0 \\ 0 & 0 & \mathcal{E} & \mathcal{E} & 0 & 1 & 0 \\ 0 & 0 & K & K & 0 & 0 & 1 \end{pmatrix}, \qquad (33)$$

$\ell = b^2[(\gamma u^2 - a^2)/Pr_T + u^2]\mu_T/(\rho R_a u N^*)$, $b_{41} = 2a^2 - \beta(I_{s1} - 2\alpha)$, $b_{42} = a^2 + 2\alpha$, $I_{s1} = h + \alpha$

A viscosity influence is taken into account only by correcting the eigenvalues in matrix D.

The matrix $|\Phi|$ is a diagonal one

$$|\Phi| = \varphi E\ ,\quad \varphi = max\{0.5 Z - 1/\Delta\xi, 0\} \qquad (34)$$

where

$$Z = \frac{1}{\bar{y}}\left(\frac{|Z_1| + \sqrt{Z_1^2 - 4Z_2}}{2}\right)\frac{N}{N^*},\quad Z_1 = \frac{(\gamma+1)uv}{N},\ Z_2 = \frac{\gamma v^2}{N}$$

In the subsonic regions the matrices $|\Omega|$ and $|\Phi|$ are diagonal ones with the corresponding elements

$$|\Omega|: \quad d_j = max\{0.5(|\omega| + 2\ell/\Delta\eta - \Delta\eta/\Delta\xi), 0\},\ \omega = |b\frac{v}{u} + \bar{a}|,$$
$$\ell = b^2 \mu_T/(\rho R_a u)$$

$$|\Phi|: \quad \varphi = v/(\bar{y}u).$$

The forth-order damping [I4] is used in the regions of shocks.

3.2. For the Eq. 25 solution the implicit method is used

$$\left(E - 0.5\Delta\xi \frac{\partial\bar{S}}{\partial\bar{F}}(\tilde{\tilde{F}}^n)\right)\left(\tilde{\tilde{F}}^{n+1} - \tilde{\tilde{F}}^n\right) = \Delta\xi\ \bar{S}(\tilde{\tilde{F}}^n), \qquad (35)$$

where the matrix $\frac{\partial\bar{S}}{\partial\bar{F}}$ has a non-zero right bottom minor 2x2 (all other elements are equal to zero); this minor has the following form

$$\frac{\partial\bar{S}}{\partial\bar{F}} = \frac{R_a}{u}\begin{bmatrix} \frac{1}{K}(-2C_{\varepsilon 2}\mathcal{E} + \Pi_k/\rho + 2\text{æ}/\rho) & \frac{\mathcal{E}}{K^2}(C_{\varepsilon 1}\mathcal{P}_T + C_{\varepsilon 2}\mathcal{E} - \text{æ}/\rho) \\ -\frac{1}{\mathcal{E}}(\mathcal{P}_T + \mathcal{E} - \text{æ}/\rho) & \frac{1}{K}(2\mathcal{P} + \Pi_k/\rho) \end{bmatrix}$$

A matrix inversion within the Eq. 35 is a trivial operation.

4. The numerical method presented here has been applied

to the solution of jets flow.

Properties along the boundaries of the jets and streamwise pressure gradient in subsonic regions were obtained as it is described in Ref.4.

The results of supersonic inviscid jet calculation are shown in figures I and 2. The jet Mach number (M_a) is 5, the external Mach number (M_e) is IO, exit angle of the nozzle (θ_a) is IO°, static pressure ratio (P_a / P_e) is IO^7 (subscript "a" refers to the nozzle exit parameters, "e" - is to the external flow parameters), ratios of specific heats are taken as constant and equal γ_a =I.3, γ_e =I.4. Here an axial static pressure variation (Fig.I) and a flow structure (Fig.2) are shown. The predictions of the present numerical method (lines) are compared with the prediction taken from the Ref. I5 (triangles). Numbers denote the following: I - bow-shock; 2 - plume interface, 3 - barrel shock, 4 - reflected shock.

As we can see the both predictions are in good agreement with each other.

The turbulent supersonic jet calculation results are shown in Fig.3. Here it is shown the flow structure calculated by the method presented here (lines) and results of the Ref.5 (triangles). The symbols are the same, with only exception - number 2 refers to the line where nondimensional excess stagnation temperature is equal to 0.5. The jet has the following chracteristics: T_e/ T_a = 0.I, p_a/ p_e =IOO, M_a= 4, M_e= 3, $\gamma_a=\gamma_e$= I.4, θ_a = 0°. Both calculation results are in a good agreement with each other, but it is necessary to take into consideration that the method described in Ref.5 requires 4 hours of computing time on a БЭСМ-6 computer for one varient and the method presnted here - 3 minutes.

The performance of the present turbulence model in predicting the decay in centerline velocity for the Mach I.37 air jet is exhibited in Fig.4 ($T_a = T_e$, $p_a = p_e$, $\theta_a = 0°$). Curve I - when $\ae$ = 0, curve 2 - with $\ae$ calculated by Eq. I9; triangles - the experimental data of Ref. IO. Obviously the computation result is in fairly close agreement with the experiment when the compressibility parameter $\ae$ is included in a turbulence model.

A comparison of the predictions (lines) and the experimental data (triangles) of Ref.I6 for an air jet is shown in Figs.5,6;(M_a= 2, p_e/ p_a= 2.5, θ_a = 8°). Fig.5 - the pitot pressure variation along the jet centerline and Fig.6 - the pitot pressure profile at $\bar{X}$ =2.5. The predictions are shown to agree quite well with the measurements.

The calculation results (lines) and the experimental data (triangles) from the Ref.4 for a cold air jet are shown in Fig.7,8. M_a= 2, p_a/ p_e = I.47, θ_a =0°. In Fig.7 a centerline static pressure variation is shown. An agreement is satisfactory one, although the presented model after $\bar{X}$ =2I forshortens the cell lengths and dampens the wave strengths prematurely. A comparison of predicted and measured [4] longitudinal turbulent intensities is given in Fig.8: I - a prediction of this work, 2 - a prediction of Ref.4, where $\overline{u''^2}/K$ is stated to be 0.96. The better agreement with the data (triangles) is obtained because of the including of the parameters $\ae$ and π_k in the turbulence model.

5. The MacCormack implicit method modification for the supersonic jets calculation on the basis of the Navier-Stokes parabolized equations has been worked out.

The results obtained according to this method are in fairly close agreement with the calculations results on the basis of other methods and experimental data.

The presented turbulence model allows to improve the agreement of the calculation results with the experimental data .

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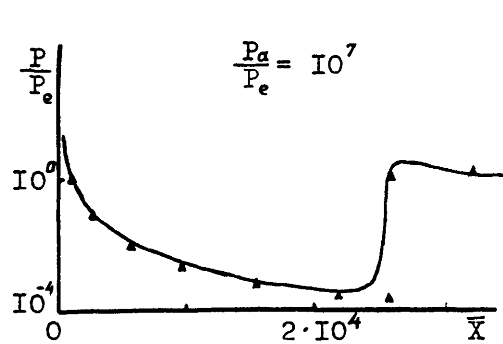

Fig.I

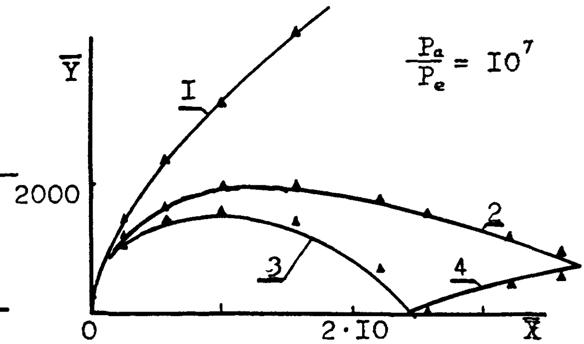

Fig.2

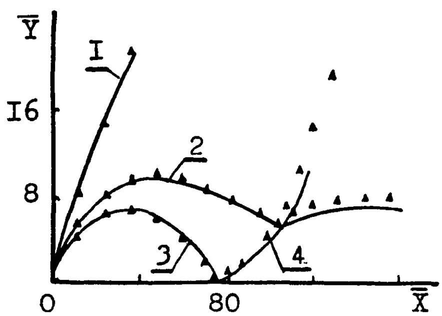

Fig.3

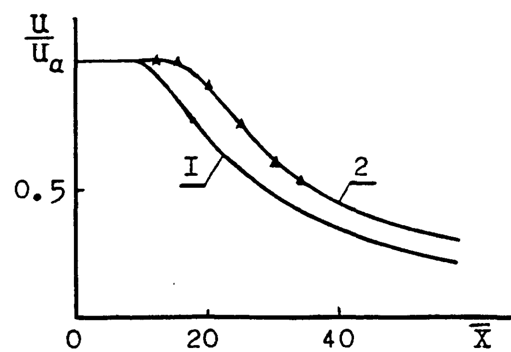

Fig.4

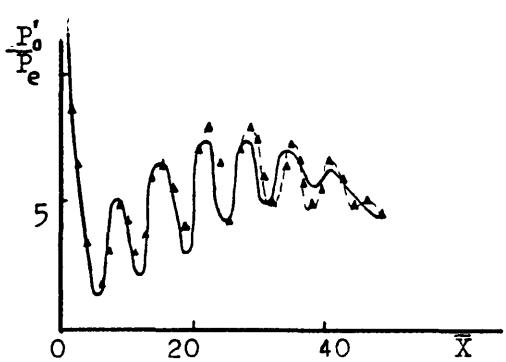

Fig.5

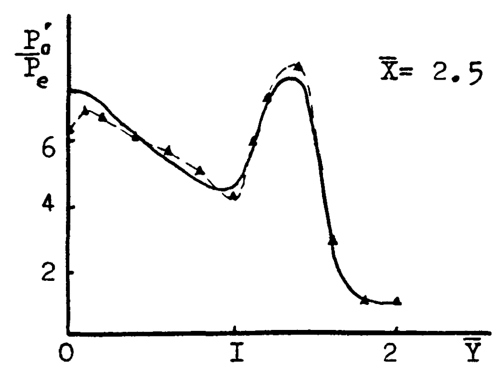

Fig.6

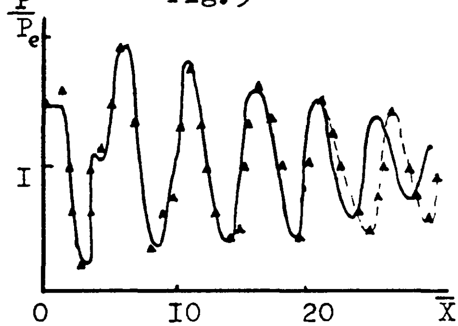

Fig.7

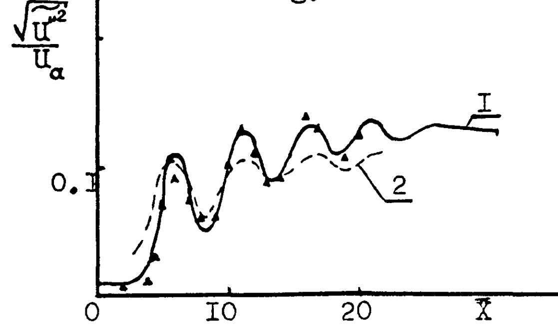

Fig.8